(2)

MRC Technical Summary Report #2821

AD-A158 190

DEVELOPMENT OF SINGULARITIES IN NONLINEAR VISCOELASTICITY

J. A. Nohel and M. Renardy

Mathematics Research Center
University of Wisconsin—Madison
610 Walnut Street
Madison, Wisconsin 53705

May 1985

(Received May 6, 1985)

DTIC FILE COPY

Approved for public release
Distribution unlimited

DTIC ELECTE AUG 2 0 1985 E

Sponsored by

U. S. Army Research Office
P. O. Box 12211
Research Triangle Park
North Carolina 27709

National Science Foundation
Washington, DC 20550

093

UNIVERSITY OF WISCONSIN - MADISON
MATHEMATICS RESEARCH CENTER

DEVELOPMENT OF SINGULARITIES IN NONLINEAR VISCOELASTICITY

J. A. Nohel and M. Renardy

Technical Summary Report #2821

May 1985

ABSTRACT

We discuss the motion of nonlinear viscoelastic materials with fading memory in one space dimension. We formulate the mathematical problem, survey results for global existence of classical solution to the initial value problem if the data are sufficiently small, and discuss in detail the development of singularities in initially smooth solutions for large data.

Accession For
NTIS GRA&I X
DTIC TAB
Unannounced
Justification
By
Distribution/
Availability Codes
Avail and/or
Dist Special
A-1

DDC
QUALITY
INSPECTED
1

AMS (MOS) Subject Classifications: 35L70, 45K05, 73F15

Key Words: nonlinear hyperbolic problems, Volterra integrodifferential equations, smooth solutions, dissipation, global existence, decay, development of singularities, shocks, viscoelastic solids, viscoelastic fluids, materials with memory.

Work Unit Number 1 - Applied Analysis

Sponsored by the United States Army under Contract No. DAAG29-80-C-0041. The work of M. Renardy was supported in part by the National Science Foundation under Grant No. MCS-8215064.

DEVELOPMENT OF SINGULARITIES IN NONLINEAR VISCOELASTICITY

J. A. Nohel and M. Renardy

1. Introduction and Discussion of Results

In this paper we discuss the motion of nonlinear viscoelastic materials with fading memory in one space dimension. ~~We~~ concentrate on viscoelastic solids and briefly remark on similar results for fluids. After formulating the mathematical problems, ~~we~~ survey results for global existence of classical solutions to the initial value problem, provided the initial data are sufficiently small. ~~We~~ then discuss in some detail the development of singularities in initially smooth solutions for large data.

We consider the longitudinal motion of a homogeneous one-dimensional body occupying an interval B in a reference configuration and having unit reference density. For simple materials, the stress σ at a material point x is a nonlinear functional of the entire history of the strain $\varepsilon = u_x$ at the same point x (here u denotes the displacement). In this paper, we confine ourselves to the following model problem, which can be motivated as a natural generalization of Boltzmann's constitutive relation for linear viscoelasticity [1] (the derivation of similar results in a variety of other models will be discussed in a later paper)

$$\sigma(x,t) = \phi(\varepsilon(x,t)) + \int_{-\infty}^{t} a'(t-\tau)\psi(\varepsilon(x,\tau))d\tau \ , \qquad (x \in B,\ -\infty < t < \infty) \ . \tag{1.1}$$

Here ϕ and ψ are given smooth functions $R \to R$ with

$$\phi(0) = \psi(0) = 0,\ \phi' > 0,\ \psi' > 0 \ , \tag{1.2}$$

and for physical reasons the relaxation function $a : [0,\infty) \to R$ is positive,

Sponsored by the United States Army under Contract No. DAAG29-80-C-0041. The work of M. Renardy was supported in part by the National Science Foundation under Grant No. MCS-8215064.

decreasing, convex, and $a' \in L^1[0,\infty)$, where $'$ denotes the derivative. The conditions on a imply that the stress relaxes as time increases and that deformations which occurred in the distant past have less influence on the present stress than those which oc- curred more recently. Since only a' occurs in the equation, we may use the normalization $a(\infty) = 0$. In the rheological literature the relaxation function a is often taken to be a finite linear combination of decaying exponentials with positive coefficients obtained by a least square fit to experimental data.

When (1.1) is substituted into the balance of linear momentum, the following integrodifferential equation for the displacement u results

$$u_{tt} = \phi(u_x)_x + a'*\psi(u_x)_x + f \quad , \quad x \in B,\ t > 0 \quad . \tag{1.3}$$

Here $*$ denotes the convolution $(\alpha*\beta)(t) = \int_0^t a(t-\tau)\beta(\tau)d\tau$, and f is the sum of an external body force and the history term $\int_{-\infty}^0 a'(t-\tau)\psi(u_x(x,\tau))_x d\tau$. An appropriate dynamical problem is to determine a smooth function $u : B \times (0,\infty) \to R$ which satisfies (1.3) together with appropriate boundary conditions if B is bounded and which at $t = 0$ satisfies prescribed initial conditions

$$u(x,0) = u_0(x),\ u_t(x,0) = u_1(x),\ x \in B$$

for certain smooth functions u_0 and u_1. To avoid technical complications, we assume in the following that $f = 0$. We restrict the discussion to the case when $B = \mathbf{R}$ and thus obtain the Cauchy problem

$$u_{tt} = \phi(u_x)_x + a'*\psi(u_x)_x \quad , \quad x \in \mathbf{R},\ t > 0 \quad , \tag{1.4}$$

$$u(x,0) = u_0(x),\ u_t(x,0) = u_1(x) \quad , \quad x \in \mathbf{R} \quad . \tag{1.5}$$

When $a' \equiv 0$ and ϕ satisfies (1.2), the body is purely elastic. In this case it is well known (see Lax [14], MacCamy and Mizel [15], Klainerman and Majda [13]), that in general the Cauchy problem (1.4), (1.5) does not have globally smooth solutions, no matter how smooth and small the initial data are

chosen. The initially smooth solution u develops singularities (shock waves) in finite time.

If $a' \not\equiv 0$ and a satisfies the sign conditions above, the fading memory term in (1.4) introduces a weak dissipation mechanism. Significant insight into the strength of this mechanism was gained by the work of Coleman and Gurtin [2], who studied the growth and decay of acceleration waves in materials with memory. They showed that the amplitude $q(t)$ of an acceleration wave propagating into a homogeneously strained medium at rest satisfies a Bernoulli-Riccati ordinary differential equation. The coefficient of q^2 in this equation is proportional to a second order elastic modulus, which is given by ϕ'' in our model problem, and there is a linear damping term proportional to $a'(0)$. Thus amplitude $q(t) = [u_{tt}]$ decays to zero as $t \to \infty$, provided $|q(0)|$ is sufficiently small. On the other hand, if $\phi'' \neq 0$, then $q(t) \to \infty$ in finite time if $|q(0)|$ is large enough, and $q(0)$ is of a certain sign.

This suggests that, under appropriate assumptions on ϕ, ψ and a, the Cauchy problem (1.4), (1.5) should have globally defined classical (C^2) solutions for sufficiently smooth and small initial data u_0, u_1, while smooth solutions should develop singularities in finite time if the initial data are large in an appropriate sense. Such a global existence result for small data was recently established by Hrusa and Nohel [10] using delicate a priori estimates obtained by combining an energy method with properties of Volterra equations (even in the presence of a small body force). We refer to a recent survey [9] for earlier small data results on initial boundary value problems modelling the motion of finite viscoelastic bodies, and for technical simplifications of the analysis in the special cases $\phi \equiv \psi$ or $a(t) = e^{-t}$. For the global results the Cauchy problem is more difficult than the finite

body problem because the Poincaré inequality is not available to estimate lower order derivatives from higher order derivatives.

The remainder of our discussion will deal with the formation of singularities in finite time from smooth solutions of the Cauchy problem (1.4), (1.5). For the special case $\phi \equiv \psi$, Markowich and Renardy [17] have obtained numerical evidence for the formation of shock fronts in finite time from large data, and Hattori [7] has shown that, if $\phi'' \not\equiv 0$ and if the body B is finite, then there are smooth initial data (which he does not characterize) for which the corresponding Dirichlet-initial value problem does not have a globally defined smooth solution. On the other hand, Hrusa [8] has shown that if ϕ is linear and only ψ is allowed to be nonlinear, then the Cauchy problem (1.4), (1.5) does have globally smooth solutions, even for large smooth data. Therefore, we shall restrict ourselves to the case when $\phi'' \neq 0$, at least over the range of the solution. The case when ϕ'' changes sign will require further refinements.

An important ingredient in the analysis (which is also important for the global theory) is the following local existence result which is established by combining Banach's fixed point theorem on an appropriate function space with standard energy estimates and Sobolev's embedding theorem.

Proposition 1:

Assume that $\phi, \psi \in C^3(\mathbb{R})$ satisfy (1.2); assume $a, a',$ $a'' \in L^1_{loc}[0,\infty)$,(*) and there is a constant $\kappa > 0$ such that

$$\phi'(\xi) \geq \kappa, \quad \xi \in \mathbb{R} \quad .$$

(*) Here the square bracket means integrability up to 0. No sign condition on a are required, but $a'(0)$ finite is essential.

Assume that $u_0 \in L^2_{\ell oc}(\mathbb{R})$ and that $u_0', u_1 \in H^2(\mathbb{R})$. Then the Cauchy problem (1.4), (1.5) has a unique classical solution $u \in C^2(\mathbb{R} \times [0,T_0))$ defined on a maximal interval $(0,T_0)$. If T_0 is finite, then

$$\sup_{\mathbb{R}\times[0,T_0)} [|u_{xx}(x,t)| + |u_{xt}(x,t)|] = \infty \quad .$$

The proof of Proposition 1 is almost identical to that of Theorem 2.1 of [6], and we omit the details; only certain readily available energy estimates for lower order derivatives are needed. The characterization of the maximal interval of existence is established by combining the energy estimates obtained in [6] with a Gronwall inequality argument. We remark that the energy estimates used in the proof of Proposition 1 yield time-dependent bounds which cannot be used to obtain global estimates. These can only be constructed by taking advantage of the damping mechanism induced by the memory term under appropriate sign conditions on a and by assuming the initial data to be small (see [10] for details).

The assumptions concerning the kernel a in Proposition 1 imply that a' is absolutely continuous on $[0,\infty)$. Recently, Hrusa and Renardy [11] established a result similar to Proposition 1 (and proved a global existence result for small data for bounded bodies) under assumptions which permit a singularity in a' at $t = 0$ (e.g. $a'(t) \sim -t^{\alpha-1}$, $0 < \alpha < 1$ as $t \to 0^+$). Such singularities are relevant for certain popular models of viscoelastic materials.

Our main result on development of singularities for large enough data is

Theorem 1:

Let $\phi, \psi \in C^3(\mathbb{R})$ satisfy (1.2) and assume $a, a', a'' \in L^1_{\ell oc}[0,\infty)$. Assume that $\phi''(0) \neq 0$. Then, for every $T_1 > 0$, we can choose initial data $u_0', u_1 \in C^2(\mathbb{R}) \cap L^\infty(\mathbb{R})$ such that the maximal time interval of

existence, given by Proposition 1, for the smooth solution of the Cauchy problem (1.4), (1.5) cannot exceed T_1. More precisely, if $\sup_{x\in\mathbb{R}} |u_0'(x)|$ and $\sup_{x\in\mathbb{R}} |u_1(x)|$ are sufficiently small, while $u_0''(x)$ and $u_1'(x)$ assume sufficiently large values with appropriate signs, then there is some $t^* < T_1$ such that

$$\sup_{\mathbb{R}\times[0,t^*)} \{|u_{xx}(x,t)| + |u_{xt}(x,t)|\} = \infty \quad , \tag{1.6}$$

while

$$\sup_{\mathbb{R}\times[0,t^*)} \{|u_x(x,t)| + |u_t(x,t)|\} < \infty \tag{1.7}$$

(and in fact, this latter quantity remains small).

In view of the analogy with hyperbolic conservation laws and the numerical evidence [17], it is to be expected that a blow-up as established by Theorem 1 will lead to the development of a shock front.

The method of the proof, sketched in section 2, is to show that the memory term is in fact of lower order than the elastic term $\phi(u_x)_x$ and can be treated as a perturbation. While considerably more technical, the proof is a generalization of the approach of Lax [14] for showing the development of singularities for the quasilinear wave equation

$$u_{tt} = \phi(u_x)_x \quad .$$

Theorem 1 was established independently by Dafermos [4] using an approach which is different from ours but similar in spirit. The result can also be established by modifying the results of F. John [12] and extending them to systems of quasilinear hyperbolic conservation laws which contain lower order source terms (F. John, private communications).

Similar results for first order model problems were derived by Malek-Mandani and Nohel [16] and, using different methods, by Renardy [18] and Dafermos [3].

A particular case of the model equation studied in this paper leads to a model for shearing flows of viscoelastic fluids studied recently by Slemrod [20]. With $v(x,t)$ denoting the velocity of the fluid in simple shear, Slemrod studies the problem

$$\begin{aligned} v_t &= a*\phi(v_x)_x \quad , \quad (x \in R,\ t > 0) \quad , \\ v(x,0) &= v_0(x) \quad , \quad (x \in R) \quad . \end{aligned} \tag{1.8}$$

for the special case $a = e^{-t}$. Problem (1.8) leads to a Cauchy problem of the form (1.4), (1.5) after differentiation with respect to time. Thus Theorem 1 can be used to get a blow-up result for this problem, like the result found by Slemrod for $a(t) = e^{-t}$. The global existence of solutions for small data follows from [5, Theorem 4.1]. Other popular models for viscoelastic fluids have been analyzed by the method used in this paper; the results will be published elsewhere.

2. Development of Shocks

In this section, we sketch the proof of Theorem 1 establishing the development of shocks from initially smooth solutions of the Cauchy problem (1.4), (1.5) in finite time. For simplicity, most of the analysis will be carried out for the special case $a(t) = e^{-t}$; the proof for more general relaxation functions as well as for a more general class of model equations will be carried out in a forthcoming paper.

We begin by transforming (1.4) to an equivalent system. We let $w = u_x$, $v = u_t$, and write the constitutive assumption (1.1) in the form

$$\sigma = \phi(w) - z \quad , \quad z = -a'*\psi(w) \quad .$$

Since we have assumed $\phi' > 0$, the first of these equations can be solved for w,

$$w = \phi^{-1}(\sigma+z) =: g(\sigma,z) \quad ,$$

and g is a smooth function of $\sigma \in \mathbf{R}$, $z \in \mathbf{R}$. As long as the solution remains smooth, the Cauchy problem (1.4), (1.5) is equivalent to the first order system

$$\begin{aligned} v_t &= \sigma_x , \\ \sigma_t &= c^2(\sigma,z)v_x + a'(0)\psi(g(\sigma,z)) + a''*\psi(g(\sigma,z)) , \\ z_t &= -a'(0)\psi(g(\sigma,z)) - a''*\psi(g(\sigma,z)) . \end{aligned} \tag{2.1}$$

The initial conditions become

$$v(x,0) = u_1(x), \ \sigma(x,0) = \phi(u_0'(x)), \ z(x,0) = 0 . \tag{2.2}$$

By c we have denoted the wave speed

$$c(\sigma,z) := [\phi'(g(\sigma,z))]^{1/2} ;$$

c is a smooth function of σ and z. The system (2.1) is hyperbolic, and its eigenvalues are $+c$, $-c$ and 0. Under the assumptions of Proposition 1, a C^1-solution exists on some maximal interval $\mathbf{R} \times [0,T_0)$. If T_0 is finite, then v, σ, z or one of their first derivatives must become infinite as $t \to T_0$. It is immediate from equation (2.1) that σ_t, σ_x, z_t and z_x will remain bounded as long as v, σ, z, v_t and v_x are bounded.

To proceed further, we extend the classical approach of Lax [14] for first order hyperbolic 2×2-systems. We define "approximate" Riemann invariants by those quantities which would be the classical Riemann invariants if z in the first two equations of (2.1) were treated as a parameter. These quantities are given by

$$\begin{aligned} r &= r(v,\sigma,z) = v + \Phi(\sigma,z) , \\ s &= s(v,\sigma,z) = v - \Phi(\sigma,z) , \\ \Phi(\sigma,z) &= \int_{\sigma_0}^{\sigma} \frac{d\zeta}{c(\zeta,z)} ; \end{aligned} \tag{2.3}$$

thout loss of generality we may take $\sigma_0 = 0$. Since $\Phi_\sigma(\sigma,z) = \frac{1}{\sigma,z)} > 0$, this correspondence is smoothly invertible, and we have

$$v = \frac{r+s}{2}, \ \Phi(\sigma,z) = \frac{r-s}{2} \quad .$$

ı the following, we assume $a(t) = e^{-t}$. Then (2.1) takes the simple form

$$\begin{aligned} v_t &= \sigma_x \quad , \\ \sigma_t &= c^2(\sigma,z)v_x - \psi(g(\sigma,z)) + z \quad , \\ z_t &= \psi(g(\sigma,z)) - z \quad . \end{aligned} \tag{2.4}$$

e now differentiate r and s along the c and $-c$ characteristics, espectively, and z along the zero characteristic (i.e. we form $_t - cr_x$, $s_t + cs_x$ and z_t). This leads to the following first order ıyperbolic system equivalent to (2.4), (2.2)

$$\begin{aligned} r_t - Ar_x &= -Bz_x + CD \quad , \\ s_t + As_x &= -Bz_x - CD \ , \\ z_t \quad\quad &= D \quad , \end{aligned} \tag{2.5}$$

ɩith the initial data

$$\begin{aligned} r(x,0) &= u_1(x) + \Phi(\phi(u_0'(x)),0) \quad , \\ s(x,0) &= u_1(x) - \Phi(\phi(u_0'(x)),0) \quad , \\ z(x,0) &= 0 \quad ; \end{aligned} \tag{2.6}$$

$$A = A(r,s,z) := c(\sigma(r,s,z),z) > 0 \quad ,$$

$$B = B(r,s,z) := c(\sigma(r,s,z),z)\Phi_z(\sigma(r,s,z),z) \quad ,$$

$$C = C(r,s,z) := \Phi_z(\sigma(r,s,z),z) - \frac{1}{c(\sigma(r,s,z),z)} \quad ,$$

$$D = D(r,s,z) := \psi(g(\sigma(r,s,z),z)) - z \quad .$$

To establish the development of shocks in finite time, we study the lution along characteristics of the quantities

$$\rho := v_x + \frac{\sigma_x}{c(\sigma,z)} \quad , \qquad \tau := v_x - \frac{\sigma_x}{c(\sigma,z)} \quad , \tag{2.8}$$

z_x. Note that if z were a constant parameter, then ρ and τ would the x-derivatives of r and s. We have $v_x = \frac{1}{2}(\rho+\tau)$, $\sigma_x = \frac{1}{2}c(\rho-\tau)$, and

$$(c^2)_\sigma(\sigma,z) = 2cc_\sigma = \frac{\phi''(g(\sigma,z))}{\phi'(g(\sigma,z))}.$$

edious but straightforward calculation using the relations (obtained by ferentiating (2.4))

$$\begin{aligned} v_{tx} &= \sigma_{xx} \quad , \\ \sigma_{tx} &= c^2(\sigma,z)v_{xx} + (c^2)_\sigma(\sigma,z)\sigma_x v_x \\ &\quad + (c^2)_z(\sigma,z)z_x v_x - D_x \quad , \\ z_{tx} &= D_x \quad , \end{aligned} \tag{2.9}$$

ds the system

$$\begin{aligned} \rho_t - c\rho_x &= \frac{(c^2)_\sigma}{4}\rho(\rho-\tau) + O(|\rho||z_x| + |\tau||z_x| \\ &\qquad + |\rho| + |\tau| + |z_x|) \quad , \\ \tau_t + c\tau_x &= -\frac{(c^2)_\sigma}{4}\tau(\rho-\tau) + O(|\rho||z_x| + |\tau||z_x| \\ &\qquad + |\rho| + |\tau| + |z_x|) \quad , \\ z_{xt} &= O(|\rho| + |\tau| + |z_x|) \quad . \end{aligned} \tag{2.10}$$

subject to the initial data

$$\begin{aligned}
\rho(x,0) &= u_1'(x) + \phi'(u_0'(x))^{1/2} u_0''(x) \quad , \\
\tau(x,0) &= u_1'(x) - \phi'(u_0'(x))^{1/2} u_0''(x) \quad , \qquad (2.11) \\
z_x(x,0) &= 0 \quad .
\end{aligned}$$

The cross product terms $\rho\tau$ in (2.10) are eliminated if one considers the characteristic derivatives of $c(\sigma,z)^{1/2}\rho$ and $c(\sigma,z)^{1/2}\tau$ (see Lax [14] and Slemrod [19]). We find

$$\begin{aligned}
&\left(\frac{\partial}{\partial t} - c\,\frac{\partial}{\partial x}\right)(c^{1/2}\rho) = \gamma(c^{1/2}\rho)^2 + O(|\rho||z_x| + |\tau||z_x| \\
&\qquad\qquad + |\rho| + |\tau| + |z_x|) \quad , \\
&\left(\frac{\partial}{\partial t} + c\,\frac{\partial}{\partial x}\right)(c^{1/2}\tau) = \gamma(c^{1/2}\tau)^2 + O(|\rho||z_x| + |\tau||z_x| \qquad (2.12) \\
&\qquad\qquad + |\rho| + |\tau| + |z_x|) \quad , \\
&z_{xt} = O(|\rho| + |\tau| + |z_x|) \quad .
\end{aligned}$$

Here the coefficient function γ is given by

$$\gamma = \gamma(\sigma,z) = \frac{1}{4}\,\frac{\phi''(g(\sigma,z))}{\phi'(g(\sigma,z))^{5/4}} \quad .$$

For definiteness, let us assume $\phi''(0) > 0$ (the discussion for $\phi''(0) < 0$ is analogous). We take initial data with the following properties: u_0' and u_1 (and hence $\rho(x,0)$, $\tau(x,0)$ as well as $z(x,0) \equiv 0$) are uniformly small, and $\rho(x,0)$, $\tau(x,0)$ are such that at least one of them has a large positive maximum (by choosing u_0'' or u_1' or both sufficiently large). At the same time, the maxima of $-\rho$ and $-\tau$ should not be too large.

As long as (r,s,z) remains within a given neighborhood U of 0, we have upper and lower bounds for the coefficients occuring in (2.12), in particular, we have a positive lower bound γ_0 for γ. We shall see later that (r,s,z) will remain in U up to the time of blow-up if they are small

enough initially and if we make the maximum of $\rho(x,0)$ or $\tau(x,0)$ large enough.

For every $t > 0$, we now set

$$h(t) = \max[\max_x \rho(x,t), \max_x \tau(x,t)] \quad .$$

From (2.12), we find that, as long as $(r,s,z) \in U$, while $h(t)$ is large and $\max_x |z_x| \ll h(t)$, we have, for some positive constants γ_0 and κ

$$\left(\frac{d}{dt}\right)_+ h(t) > \gamma_0 (h(t))^2, \quad \text{and} \quad \max_x |z_{xt}| < \kappa h(t) \ll (h(t))^2 \quad .$$

Initially, we have $|z_x| = 0$ and it follows from these inequalities that it will remain small compared to $h(t)$. We also find that $h(t)$ becomes infinite in finite time. Since there is also some constant γ_1 such that $\left(\frac{d}{dt}\right)_+ h(t) < \gamma_1 (h(t))^2$, it can be shown that, with t^* denoting the blow-up time of h, we have $\frac{c_1}{t^*-t} < h(t) < \frac{c_2}{t^*-t}$ for some constants c_1 and c_2. The third equation of (2.12) then implies that $|z_x|$ grows at most logarithmically as $t \to t^*$. Since $\log(t^*-t)$ is integrable, equations (2.5) imply that r, s, and z remain bounded and in fact small if their initial data are small, and t^* is small (which is the case if $h(0)$ is large). In this way, we can choose the data such that (r,s,z) will in fact remain in U up to the time of blow-up. This completes the sketch of the proof.

References:

[1] L. Boltzmann, Zur Theorie der elastischen Nachwirkung, Ann. Phys. 7 (1876), Ergänzungsband, 624-654.

[2] B. D. Coleman, M. E. Gurtin and I. R. Herrera, Waves in materials with memory, Arch. Rat. Mech. Anal. 19 (1965), 1-19; B. D. Coleman and M. E. Gurtin, ibid, 239-265.

[3] C. M. Dafermos, Dissipation in materials with memory, in: J. A. Nohel, M. Renardy and A. S. Lodge (eds.), Viscoelasticity and Rheology, Academic Press, to appear.

[4] C. M. Dafermos, Development of singularities in the motion of materials with fading memory, Arch. Rat. Mech. Anal., to appear.

[5] C. M. Dafermos and J. A. Nohel, Energy methods for nonlinear, hyperbolic Volterra integrodifferential equations, Comm. PDE 4 (1979), 219-278.

[6] C. M. Dafermos and J. A. Nohel, A nonlinear hyperbolic Volterra equation in viscoelasticity, Amer. J. Math., Supplement (1981), 87-116.

[7] H. Hattori, Breakdown of smooth solutions in dissipative nonlinear hyperbolic equations, Q. Appl. Math. 40 (1982/83), 113-127.

[8] W. J. Hrusa, Global existence and asymptotic stability for a semilinear hyperbolic Volterra equation with large initial data, SIAM J. Math. Anal. 16 (1985), 110-134.

[9] W. J. Hrusa and J. A. Nohel, Global existence and asymptotics in one-dimensional nonlinear viscoelasticity, in: P. G. Ciarlet and M. Roseau (eds.), Trends and Applications of Pure Mathematics to Mechanics, Springer Lecture Notes in Physics 195 (1984), 165-187.

[10] W. J. Hrusa and J. A. Nohel, The Cauchy problem in one-dimensional nonlinear viscoelasticity, J. Diff. Eq., to appear.

[11] W. J. Hrusa and M. Renardy, On a class of quasilinear partial integro-differential equations with singular kernels, J. Diff. Eq., to appear.

[12] F. John, Formation of singularities in one-dimensional nonlinear wave propagation, Comm. Pure Appl. Math. 27 (1974), 377-405.

[13] S. Klainerman and A. Majda, Formation of singularities for wave equations including the nonlinear vibrating string, Comm. Pure Appl. Math. 33 (1980), 241-263.

[14] P. D. Lax, Development of singularities of solutions of nonlinear hyperbolic partial differential equations, J. Math. Phys. 5 (1964), 611-613.

[15] R. C. MacCamy and V. J. Mizel, Existence and nonexistence in the large for solutions of quasilinear wave equations, Arch. Rat. Mech. Anal. 25 (1967), 299-320.

[16] R. Malek-Madani and J. A. Nohel, Formation of singularities for a conservation law with memory, SIAM J. Math. Anal. 16 (1985), 530-540.

[17] P. A. Markowich and M. Renardy, Lax-Wendroff methods for hyperbolic history value problems, SIAM J. Num. Anal. 21 (1984), 24-51; Corrigendum, SIAM J. Num. Anal. 22 (1985), 204.

[18] M. Renardy, Recent developments and open prolems in the mathematical theory of viscoelasticity, in: J. A. Nohel, M. Renardy and A. S. Lodge (eds.), <u>Viscoelasticity and Rheology</u>, Academic Press, to appear.

[19] M. Slemrod, Instability of steady shearing flows in a nonlinear viscoelastic fluid, Arch. Rat. Mech. Anal. 68 (1978), 211-225.

[20] M. Slemrod, Appendix: Breakdown of smooth shearing flow in visco-elastic fluids for two constitutive relations: the vortex sheet vs. the vortex shock, in: D. D. Joseph, Hyperbolic phenomena in the flow of viscoelastic fluids, to appear in: J. A. Nohel, M. Renardy and A. S. Lodge (eds.), <u>Viscoelasticity and Rheology</u>, Academic Press.

JAN/MR/jvs

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER #2821	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Development of Singularities in Nonlinear Viscoelasticity		5. TYPE OF REPORT & PERIOD COVERED Summary Report - no specific reporting period 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) J. A. Nohel and M. Renardy		8. CONTRACT OR GRANT NUMBER(s) MCS-8215064 DAAG29-80-C-0041
9. PERFORMING ORGANIZATION NAME AND ADDRESS Mathematics Research Center, University of Wisconsin 610 Walnut Street Madison, Wisconsin 53705		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Work Unit Number 1 - Applied Analysis
11. CONTROLLING OFFICE NAME AND ADDRESS See Item 18 below		12. REPORT DATE May 1985 13. NUMBER OF PAGES 14
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

U. S. Army Research Office
P. O. Box 12211
Research Triangle Park
North Carolina 27709

National Science Foundation
Washington, DC 20550

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

nonlinear hyperbolic problems, Volterra integrodifferential equations, smooth solutions, dissipation, global existence, decay, development of singularities, shocks, viscoelastic solids, viscoelastic fluids, materials with memory.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

We discuss the motion of nonlinear viscoelastic materials with fading memory in one space dimension. We formulate the mathematical problem, survey results for global existence of classifical solution to the initial value problem if the data are sufficiently small, and discuss in detail the development of singularities in initially smooth solutions for large data.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)